large number of well graded applications that have been included. The solving of these applications cannot fail to give the student a better understanding of the fundamentals involved and at the same time should tend to stimulate his interest in the subject.

WILLIAM A. JOHNSTON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

PROBLEMS FOR SOLUTION.

ALGEBRA.

438. Proposed by WALTER C. EELLS, U. S. Naval Academy, Annapolis, Maryland.

In Hardy's *Pure Mathematics* (page 14, Nos. 2, 3) occurs the problem: "Show that if m/n is a good approximation to $\sqrt{2}$, then $(m+2n)/(m+n)$ is a better one, and that the errors in the two cases are in opposite directions, *e. g.*, 1/1, 3/2, 7/5, 17/12, 41/29, 99/70, $\cdots$." Find (a) other approximations for $\sqrt{2}$ of same type, *i. e.*,

$$\frac{m'}{n'}=\frac{am+bn}{cm+dn}, \qquad (a,\ b,\ c,\ d,\ m,\ n,\ \text{integers}).$$

(b) Similar approximations for the square roots of other integers.

439. Proposed by A. M. KENYON, Purdue University.

If k, n are natural numbers, $n > 2k$, show that

$$\frac{2^k}{\underline{|k}}\sum_{i=0}^{I\left(\frac{n+1}{2}\right)}\frac{1}{\underline{|2i+1}\,\underline{|n-k-2i}}=\frac{2^n}{\underline{|n+1}}\sum_{i=0}^{k}\binom{n-i}{n-k},$$

where $I(n/2)$ denotes the integral part of $n/2$ and $\binom{n}{k}$ is the coefficient of x^k in $(1+x)^n$.

440. Proposed by W. D. CAIRNS, Oberlin College.

n being a positive integer, find the sum of the series

$$n^2+4(n-1)^2+2(n-2)^2+4(n-3)^2+2(n-4)^2+\cdots, \tag{1}$$

where the succeeding coefficients are alternately 4 and 2; or, more generally, the series

$$an^2+b(n-1)^2+a(n-2)^2+b(n-2)^2+b(n-3)^2+\cdots. \tag{2}$$

L'Intermédiaire, July, 1913.

GEOMETRY.

469. Proposed by W. F. FLEMING, Chicago, Ill.

A pole whose length is l stands vertically against a vertical wall. A spider is at each end of the pole. The pole is drawn out from the wall in such a way that its upper end moves down the wall at a uniform rate. At the same time that the pole begins to move, the spiders begin to travel toward each other at rates equal to the rates at which the respective ends move. Determine the equations of the paths of the two spiders, in space.

470. Proposed by ROBERT E. MORITZ, University of Washington.

Prove that

$$\theta=\left(\lambda+\frac{q}{p}\mu\right)\pi, \qquad (\lambda=1,\ 2,\ 3,\ \cdots,\ q-1;\ \mu=0,\ 1,\ 2,\ \cdots,\ p-1)$$

and

$$\theta = (2\lambda - 1)\frac{\pi}{2} + \frac{q}{p}(4\mu \pm 1)\frac{\pi}{2}, \qquad (\lambda = 1, 2, 3, \cdots, \frac{q-1}{2};\ \mu = 0, 1, 2, \cdots, p-1);$$

determine the same set of points on the curve

$$\rho = a \cos \frac{p}{q}\theta,$$

where p and q are two odd integers without a common factor, and a is any constant.

471. Proposed by C. N. SCHMALL, New York City.

In the ellipse $x^2/a^2 + y^2/b^2 = 1$, an *equilateral* hexagon is inscribed with two sides parallel to the major axis. In the major auxiliary circle the same thing is done. If H_1 and H_2 be the sides of the hexagons, and e the eccentricity of the ellipse, show that $H_1 : H_2 :: 4 - 2e^2 : 4 - e^2$

CALCULUS.

390. Proposed by WILSON L. MISER, University of Minnesota.

Show that the triangle whose area is a constant and whose perimeter is a minimum is equilateral.

391. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $0 < \lambda < 1$ and $0 < x < \pi$, show that the function $(\sin \lambda x)/(\sin x)$ increases as x increases.

392. Proposed by HORACE OLSON, Student at The University of Chicago.

Two right circular cylinders of radii a and b respectively, are placed so that their axes intersect at right angles. Find the volume common to them.

MECHANICS.

314. Proposed by C. N. SCHMALL, New York City.

A rectangular box of height h, and having a plane mirror for its bottom, contains a quantity of water of unknown height x. In the lid are two small apertures distant $2a$ from each other. A ray of light entering one aperture with an angle of incidence ϕ, emerges, after refraction and reflection, through the other aperture. If μ be the index of refraction of water, show that the height of the water is

$$x = \frac{(h \tan \phi - a)}{\left[\tan \phi - \dfrac{\sin \phi}{(\mu^2 - \sin^2 \phi)^{\frac{1}{2}}}\right]}.$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

427. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $r \sin(\theta + \alpha) = m$, and $r \cos(\theta + \beta) = n$, show that

$$r = \frac{\sqrt{m^2 + n^2 - 2mn \sin(\alpha - \beta)}}{\cos(\alpha - \beta)}.$$

SOLUTION BY J. H. KELLOGG, Oberlin College.

From trigonometry, using only the positive square root, we know that